

# Risk vs Greed in Evolutionary Game Theory

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The notion of risk awareness was implemented for the first time in the standard replicator equations. The system is analysed for a general  $2 \times 2$  game and a non-zero-sum rock-paper-scissors, both in a well-mixed regime. At critical values of the risk parameter, bifurcations occur and new stable fixed points appear. These have a dramatic effect on the population dynamics and the fixation probabilities, reversing the expected behaviour. A unified framework of algorithms, developed for a general n-dimensional game, capable of deterministic and stochastic simulations with and without risk, is also presented.

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## 1. Introduction

This report considers effects of risk and greed in the evolution of a population. The standard payoff based replicator system was extended to include a notion of risk awareness. To develop an understanding of this modification, the tools of nonlinear dynamics, nonequilibrium statistical mechanics and evolutionary game theory were used. This novel risk based approach, previously not seen in the literature, produces a new and rich evolutionary process. A reversal of the standard dynamics due to risk is reported for the first time in games with two and three pure strategies in well mixed populations.

## 1.1 Nonlinear Dynamical Systems

The study of dynamical systems consists of investigating the time evolution of the variables of the system which intrinsically depends on their mutual interactions. The subject is usually associated with classical mechanics but its applications can be extended to fields of chemistry, biology, sociology and economics. Here, the variables are no longer positions and momenta of particles but can represent quantities such as compound concentrations, a number of infected cells or the fraction of agents buying a certain asset. Usually, the values of these variables and their time derivatives are linked in a system of nonlinear differential equations. These are rarely solvable analytically, however, many techniques have been developed to gain an understanding of their dynamics. It is often possible to describe the system qualitatively and, what is most interesting, predict its state after a long time.<sup>[1]</sup>

## 1.2 Systems far from Equilibrium

Equilibrium statistical mechanics considers systems with infinite number of interacting particles. This is often an excellent approximation in most physical problems concerning a particle number of order  $10^{23}$ . However, there are examples where the size of the system is nowhere near the Avogadro's number. One such entity would be an *E.coli* bacterium, consisting of  $\sim 1000$  genes, each made up of  $\sim 1000$  proteins. Such systems exhibit a large degree of randomness and the thermodynamic limit fails to describe their behaviour. However, stochastic treatment of nonequilibrium statistical mechanics proves to be very appropriate in such cases. Moreover, the approach not only provides a better approximation of the dynamics but also allows to discover new phenomena, never to be predicted by the formulations in the infinite limit.<sup>[2,3]</sup>

### 1.3 Evolution and Game Theory

Evolution arises naturally whenever information is being reproduced. The two main ingredients of the processes are selection and mutation. The rates at which different pieces of information are being copied leads to the former while errors in the reproduction make up the latter. The nature of the information can be of many origins, usually one thinks of genes and DNA in the context of evolution but the process can also be applied, for instance, to software development. The history of operating systems resembles closely an evolutionary process and more examples can be found across many other disciplines. In any case, the genes, organisms or computer programs, replicate to produce copies of themselves. As there are always more than one species considered, this forms a coupled replicator system. The growth of one type of an individual often depends on interactions with other species and their relative abundances. The theory of games is used to bring those systems into a mathematical framework to simplify the analysis and allow for new insights.

Game theory was initially developed as a tool in economics. Nowadays, it is used more and more frequently to model evolutionary processes. One of the assumptions made by the theory in its economic use considers rationality of human agents and it has proven to be somewhat controversial. Here, this rationality is replaced by evolutionary stability which seems like a more plausible approximation. The usefulness of game theory comes from its treatment of interactions between the players. One can imagine designing a perfect wing or a fin to allow an animal to escape from its predator, however, a greater advantage can be achieved through, for example, dispersal behaviour adopted by certain types of birds or fish. Also, equally fit agents should not be treated equivalently by an evolutionary process due to their history. The notion of learning is perhaps more important than a simple genetic inheritance and interactions with previous generations should not be ignored as well.<sup>[4,5]</sup>

## 2. Games with 2 Strategies (2D)

### 2.1 Real-Life Examples in 2D

In this section, some examples of games with two strategies are introduced along with real life situations which closely resemble their dynamics. One of the most commonly known example is the Prisoner's Dilemma which can be met in any introductory text on the theory of games.

In this game, two individuals are alleged to commit a crime. As they have actually done so, they need to decide on a strategy which secures them the least severe punishment. In the light of the partial evidence provided by the forensic investigation, they will both serve a prison sentence of two years, which will be denoted as a payoff of -2. However, if some additional information is supplied, showing that they are indeed guilty of the crime, the sentence will be extended to five years (payoff of -5). Such evidence could be obtained by interrogating the suspects and a reduction of the prison sentence is offered as a reward for supplying the information. The accused will spend two years less in prison if he or she helps to prosecute the other criminal. However, if both suspects decide to "talk", the act is not valued as much and the sentence is reduced by a period of only one year. The individuals are separated and do not get an opportunity to discuss their strategies. They can either cooperate (C) with each other and stay quiet or defect (D) to reduce their sentences. The strategies are summarised in Table 1 below. The criminal chooses an option from one of the rows and then receives a payoff from a column corresponding to the strategy of the second criminal.

	Cooperate	Defect
Cooperate	-2	-5
Defect	0	-4

Table 1: Punishment in Prisoner's Dilemma.

It is clear that the best option for both suspects is to cooperate with each other and serve the two year sentence. However, if one of the prisoners defects, he or she will be released while the other will be detained for the full five year sentence. In this case the second criminal should also defect to gain one year reduction in the punishment. It is indeed the case, as will be proven later, that all individuals choose to defect and no one cooperates.

A similar situation arose in 1960's and 1970's with the appearance of the first warnings about cigarettes being hazardous to public health. The tobacco companies, fearing a stream of devastating court rulings, made an agreement with the US government which guaranteed them immunity from any lawsuits. In exchange, warning labels were to be placed on cigarette packs and no TV advertising was allowed. Although this seems a fair deal, one should look more closely at the situation through the prism of game theory.

Table 2 summarises payoffs for a tobacco company and as before, the strategy needs to be chosen from one of the rows and the outcome depends on the column related to the tactic of the opponent.